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Interferential polychromatic filters based on the quasi-periodic one-dimensional generalized multilayer Thue–Morse structures

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In this paper, a new type of optical filter using photonic band gap materials has been proposed. The optical filter was obtained by a combination of periodic $H(LH)^J$ and generalized Thue–Morse (GTM) quasi-periodic one-dimensional multilayer. We show that the whole structure $H(LH)^J[\mathrm{TM}]^P H(LH)^J$, where P is the repetition number of the GTM stack, has an interesting application as a polychromatic filter. However, the effect of the repetition number for producing an improved polychromatic filter is presented. So, switches like (on–on–off–off–on–on– …) have been found.

Keywords: photonic crystals, periodic, quasi-periodic, Thue–Morse, antitrace map, transmission peak, polychromatic filters.

1. Introduction

During the past few years, photonic crystal (PC) structures based in dielectric materials, received noticeably increased attention [1]. The significant optical characteristics of such structures make it possible to create the essential miniaturized photonic devices [2] which are the most promising elements of an optical digital computer [3].

Among the potential solutions of PCs devices is the filter based on PCs which can be used in wavelength division multiplexing applications.

In recent years, there has been much interest in the physics and applications of one-dimensional spatially periodic, quasi-periodic and random photonic bandgap (PBG) structures [4–7]. Quasi-periodic systems can be considered as suitable models to describe the transition from the perfect periodic structure [8].

A polychromatic filter is a device which realizes a selection of frequencies while preserving (pass-band) or rejecting (stop-band) several frequency ranges.

In order to improve the performances of the photonic structures, many studies were carried out on the polychromatic filters [9–11].

In this work, we study the polychromatic filters designed as a combination of periodic and quasi-periodic multilayers (GTM). We report a numerical simulation of the transmission proprieties of multilayer films built according to the generalized Thue−Morse sequence. We will show that the juxtaposition of periodic and quasi-periodic structure which is built according to the pattern of the Thue−Morse sequence has an interesting application as a polychromatic filter.

2. Model and formalism

2.1. The generalized Thue−Morse (GTM) one-dimensional structure

The Thue−Morse (TM) one-dimensional structure is constituted by a sequence of two layers A and B with refractive index n_A (n_L) and thickness d_A (d_B). Applying the substitution rules $A \to AB$, $B \to BA$ [12], we can deduce all subsequent orders of the sequence S_k, where k is the order.

The dielectric multi-layer structures with distribution of generalized Thue−Morse (GTM) are the generalizations of the standard structures (TM), where one define two parameters n and m, with the substitution rule $A \to A^n B^m$, $B \to B^m A^n$ [13]. These structures can be defined as GTM(n, m, k) where n and m are tow parameters of this sequence, and k the order of GTM sequence:

$$\begin{aligned} &S_1 = H \\ &\overline{S}_1 = L \\ &S_{k+1} = (S_k)^n (\overline{S}_k)^m \\ &\overline{S}_{k+1} = (\overline{S}_k)^m (S_k)^n \end{aligned} \tag{1}$$

with H and L indicate high and low indexes, respectively, and $(n, m) \in$ INXIN, $k \geq 2$.

T a b l e. 1. Generation of the quasi-periodic sequence according to GTM, $n = 1$ and $m = 2$.

S_1	*H*
S_2	*HLL*
S_3	*HLLLLHLLH*
S_4	*HLLLLHLLHLLHLLHHLLLLHLLHHLL*

T a b l e. 2. Generation of the periodic sequence.

S_1	*H*
S_2	*HLHLH*
S_3	*HLHLHLH*
S_4	*HLHLHLHLH*

According to the inflation rules of GTM sequence generation, one can readily obtain the order k of S_k sequence with $n = 1$ and $m = 2$, as shown in Tab. 1.

2.2. The periodic structure

The periodic structure, both in geometry and manufacturing, is a one-dimensional stack of two types of layers which differ in the refractive indexes n_H and n_L distributed according to the following distribution: $S_J \equiv H(LH)^J$ [10], where H and L indicate materials with high and low indexes, respectively, and J is the order (or repetition number) of this sequence. An example of generation of this structure is presented in Tab. 2.

2.3. Theory

Here, we consider one-dimensional quasi-periodic multilayer generated by the distribution of GTM. The latter is embedded between tow periodic structures, which constitute a Fabry–Perot resonator of Fig. 1.

The calculation principle of the optical transmission is based on the trace-antitrace method (for more details, see [14–16]), where the transfer matrices are given by:

$$
\begin{aligned}
P_{(\mathrm{air}/L)} &= P^{-1}_{(L/\mathrm{air})} = \begin{pmatrix} 1 & 0 \\ 0 & \dfrac{1}{n_L} \end{pmatrix} \\
P_L &= \begin{pmatrix} \cos(\delta_L) & -\sin(\delta_L) \\ \sin(\delta_L) & \cos(\delta_L) \end{pmatrix} \\
P_{(\mathrm{air}/H)} &= P^{-1}_{(H/\mathrm{air})} = \begin{pmatrix} 1 & 0 \\ 0 & \dfrac{1}{n_H} \end{pmatrix} \\
P_H &= \begin{pmatrix} \cos(\delta_H) & -\sin(\delta_H) \\ \sin(\delta_H) & \cos(\delta_H) \end{pmatrix}
\end{aligned}
\tag{2}
$$

The $P_{(\mathrm{air}/L)}$ ($P_{(L/\mathrm{air})}$) stands for the propagation matrix from the air (L) to L (the air) and P_L is the propagation matrix through the single layer L. In the same way, $P_{(\mathrm{air}/H)}$ ($P_{(H/\mathrm{air})}$) stands for the propagation matrix from the air (H) to H (the air) and P_H is the propagation matrix through the single layer H. $\delta_{H(L)} = k_{nH(L)} d_{H(L)}$, $n_{H(L)}$ is the refractive index of media $H(L)$, $d_{H(L)}$ is the layer thicknesses, and k is the wave number in vacuum. Generally, we choose appropriate layer thicknesses d_H and d_L to make $n_H d_H = n_L d_L$. Then, we have $\delta_L = \delta_H = \delta$.

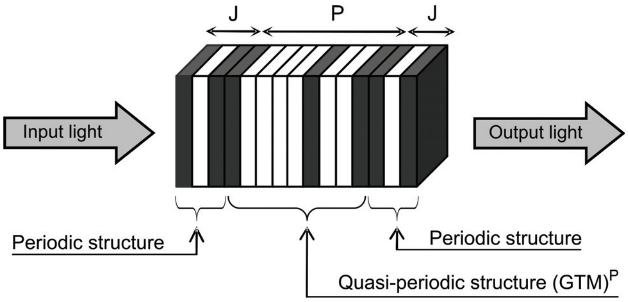


Fig. 1. The study model of configuration $H(LH)^J(H4LH2LH)^PH(LH)^J$.

The matrix systems for the photonic structure (Fig. 1) becomes:

a) for the periodic sequence:

$$P_J = P_H(P_L P_H)^J$$

with J being the repetition number of the periodic sequence;

b) for the quasi-periodic sequence $(\mathrm{GTM})^P$:

$$\begin{aligned} P_1 &= [P_H]^P \\ \overline{P}_1 &= [P_L]^P \\ P_{k+1} &= [(P_k)^n(\overline{P}_k)^m]^P \\ \overline{P}_{k+1} &= [(\overline{P}_k)^m(P_k)^n]^P \end{aligned} \tag{3}$$

with $(n, m) \in INXIN$: Tow parameters of GTM; $k \geq 2$, $k \in IN^*$: the order of the GTM sequence; $P \in IN$: the repetition number of quasi-periodic sequence.

The transmission coefficient for the whole system (Fig. 1) is obtained as:

$$T_k = \frac{4}{|P_k| + 2} \tag{4}$$

where $|P_k|^2$ is the sum of squares of the four elements of the matrix S_k. Since the transfer matrix is unimodular, the transmission coefficient can be written as:

$$T_k = \frac{4}{x_k^2 + y_k^2} \tag{5}$$

where x_k and y_k denote the trace and antitrace of the transfer matrix P_k, respectively.

Based on the trace-antitrace method we make an iterative algorithm calculation which enables us to determine the transmission spectra of the structures under study.

3. Simulation results

We study the configurations: a multiple of a two-stage generalized Thue–Morse $[\mathrm{GTM}(1,2,3)]^P$ with $\mathrm{GTM}(1,2,3) = (H3LH2LH)^P$, a juxtaposition of a periodic multilayer structure $H(LH)^J$ and a multiple of a two-stage generalized Thue–Morse multilayer $(H3LH2LH)^P$ embedded between two periodic stacks, that is, $H(LH)^J(H3LH2LH)^P\, H(LH)^J$, as shown in Fig. 1.

In the following numerical investigation, we chose SiO_2 (L) and TiO_2 (H) as two elementary layers, with refractives indexes $n_L = 1.45$ and $n_H = 2.30$ at 700 nm.

Firstly, we study the structure GTM(1,2,3) for $P = 1$, *i.e.*, $n = 1$, $m = 2$ for the iteration $k = 3$ and for $P = 1$. The transmission spectrum of this structure (Fig. 2) shows the following characteristics:

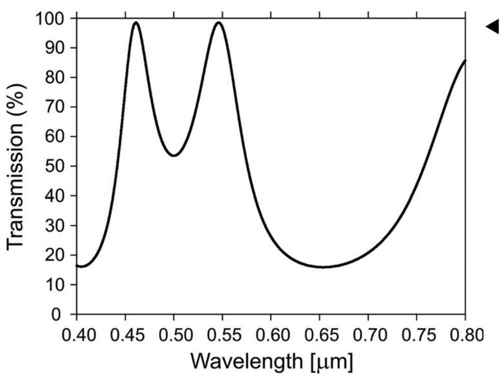
– distribution of the layers ($HLLLLHLLH$);
– the number of high index layers is 3;
– the number of low index layers is 6;
– the spectrum presents two peaks of transmission ($\lambda_{\mathrm{peak1}} = 0.4611$ μm, $\lambda_{\mathrm{peak2}} = 0.5461$ μm), with a transmission value for $\lambda_0 = 0.5$ μm equal to 53.4828%.

3.1. Juxtaposition of *P* generalized Thue–Morse distribution multi-layer systems

In this part, we are interested in the study of the multilayer system following $(S)^P$, where S is a multi-layer system built according to the distribution of GTM, with $n = 1$, $m = 2$ for the third iteration.

We thus juxtaposed P systems GTM(1,2,3) in order to study the optical characteristics of this dielectric multilayer structure in a spectral range [0.4 μm, 0.6 μm] and under normal incidence (Fig. 3).

The transmission spectra of system $(H3LH2LH)^P$ with the repetition number P varying from 2 to 10 are shown in Fig. 4.



◄ Fig. 2

▼ Fig. 3

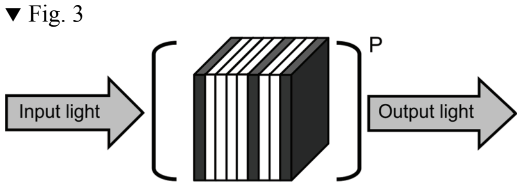


Fig. 2. Spectrum of transmission of the multilayer structure GTM(1,2,3) in the visible range.

Fig. 3. The model of configuration $(H4LH2LH)^P$.

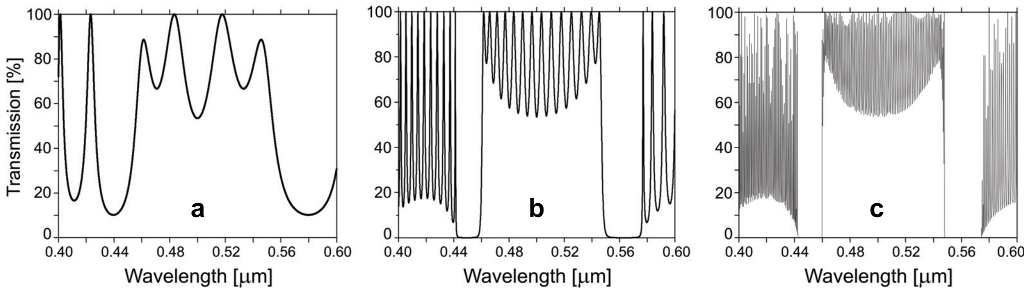


Fig. 4. Transmission versus wavelength for the $(H4LH2LH)^P$ structure for different P values: 2 (**a**), 5 (**b**), and 10 (**c**).

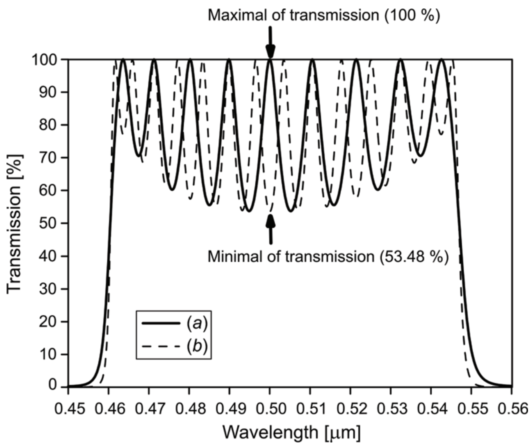


Fig. 5. Transmission versus wavelength for: $(H4LH2LH)^4$ (*a*) and $(H4LH2LH)^5$ (*b*).

According to the transmission spectra, it is clear that if P varies, the transmission for $\lambda_0 = 0.5$ μm shows a characteristic of switches. Indeed, one notes by ON a maximum of transmission (100%) and by OFF the minimum (53.48%) for $\lambda = \lambda_0 = 0.5$ μm, the following characteristics, as shown in Fig. 5 by zooming the spectra of Figs. 4**b** and 4**c** in the spectral range 0.45–0.56 μm: S_0(ON)–S_1(OFF)–S_2(OFF)–S_3(ON)–S_4(ON)–S_5(OFF)–S_6(OFF)–S_7(ON)–S_8(ON)–S_9(OFF)...

This characteristic can be modeled by the following relations:

$$\begin{cases} S_0(\mathrm{ON}) & \text{if} \quad P = 0 \, \forall n \in \mathrm{IN} \\ S_P(\mathrm{OFF}) & \text{if} \quad P = 1 + 4n \quad \text{or} \quad P = 2 + 4n \\ S_P(\mathrm{ON}) & \text{if} \quad P = 3 + 4n \quad \text{or} \quad P = 4 + 4n \end{cases} \tag{6}$$

The number of the peaks around $\lambda_0 = 0.5$ μm can be represented in a polynomial way versus P according to the following relation:

$$N_{\mathrm{peak}}(P) = 0.5(P^2 + P) - 1 \quad \forall \ P \geq 2 \tag{7}$$

3.2. The optimization of the number *J* of the PMS

In order to optimize the repetitive number J of the periodic multilayer structure $H(LH)^J$ we study under normal incidence the transmission spectra (Fig. 6) in the spectral range [0.44 μm, 0.57 μm] of the system $H(LH)^J(H3LH2LH)^P H(LH)^J$. Subsequently, we fixed P to 3 and let J vary from 0 to 8.

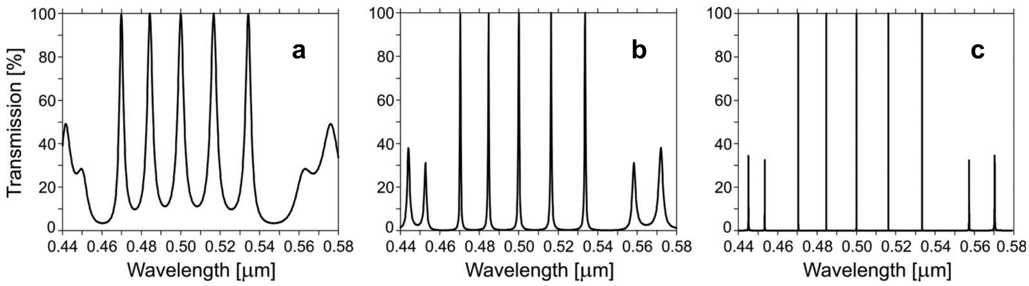


Fig. 6. Transmission spectrum for the $H(LH)^J(H4LH2LH)^3H(LH)^J$ structure for: $J = 1$ (**a**), $P = 3$ (**b**), $J = 8$ (**c**).

We notice that the transmission spectra present the same number of peaks. This number is fixed to 5. Moreover, we note that:

– The increase of J induces a reduction in full width at half maximum (FWHM) for the five peaks as shown in Fig. 7;

– The minima of the transmission peaks $T_{\min\%}(0.5\ \mu\text{m})$ decrease to 0% by increasing J (Fig. 8).

Starting from $J = 4$ at $\lambda = 0.5$ μm, the FWHM decrease in a very weak way (Fig. 7). On the other hand, the minima of transmission $T_{\min\%}(0.5\ \mu\text{m})$ decrease in an important way (Fig. 8).

We can conclude that the optimal value J which can be chosen for the configuration $H(LH)^J(H4LH2LH)^3H(LH)^J$ is $J \geq 4$. The value $J = 4$ corresponds to the minimal periodical (LH) number which can be traced to have minimal transmission $T_{\min\%}(0.5\ \mu\text{m}) = 0.05\%$ and FWHM $= 1.8\times10^{-4}$ μm. Indeed, from $J = 4$ no considerable improvement in the performance of the systems is observed compared with

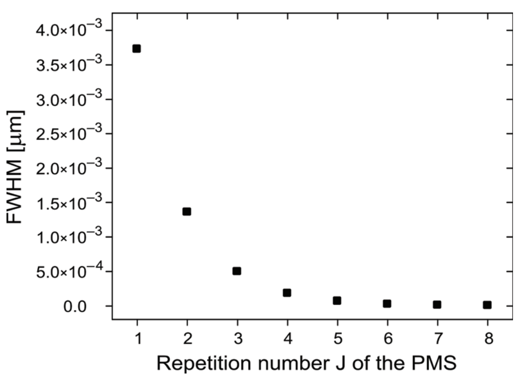


Fig. 7. Plot of the average of FWHM of the peaks versus the repetition number J of $H(LH)^J(H3LH2LH)^3H(LH)^J$.

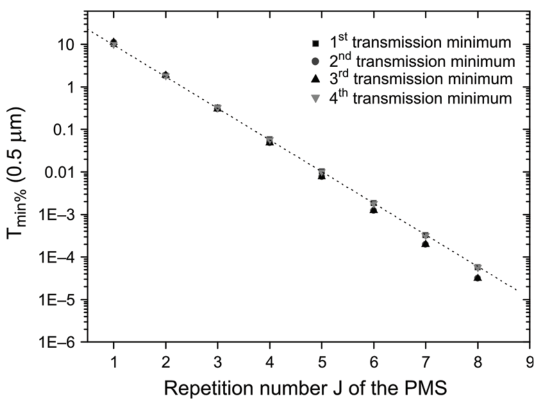


Fig. 8. Presentation on a logarithmic scale of the minima of transmission $T_{\min\%}$ in the vicinity of 0.5 μm for $\lambda_1 = 0.4759$ μm, $\lambda_1 = 0.4917$ μm, $\lambda_1 = 0.5086$ μm and $\lambda_1 = 0.5266$ μm.

the cases of $J = 1$, $J = 2$, and $J = 3$. Consequently, we choose the value of $J = 4$ which corresponds to acceptable system performances with minimal values of layers as described elsewhere.

3.3. Variation effect of the repetitive number *P*

For the optimal value $J = 4$, we let P vary from 0 to 8 and we study the transmission spectra (Fig. 9) for the configuration $H(LH)^4(H4LH2LH)^P H(LH)^4$.

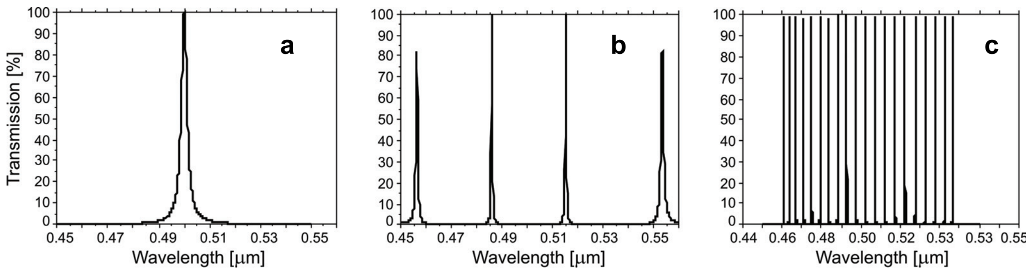


Fig. 9. Transmission spectrum for the $H(LH)^4(H3LH2LH)^P H(LH)^4$ structure for: $P = 0$ (**a**), $P = 2$ (**b**), $P = 5$ (**c**).

According to the results obtained, we notice that the transmission presents a multitude of peaks around λ_0, where their number obeys a polynomial behavior versus P which can be written in the form (7).

In parallel, we followed the peak positions versus P in the spectral range form 0.45 to 0.56 μm as shown in Fig. 10.

For the value $\lambda_0 = 0.5$ μm, the transmission follows the switch like properties: S_0(ON)–S_1(OFF)–S_2(OFF)–S_3(ON)–S_4(ON)–S_5(OFF)–S_6(OFF)–S_7(ON)–S_8(ON)––S_9(OFF)… which again obey the properties (6).

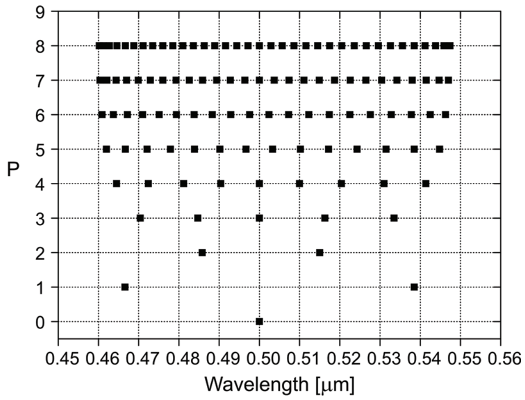


Fig. 10. Position of the transmission peaks for structures $H(LH)^4[\mathrm{GTM}(1,2,3)]^P H(LH)^4$, with P varying from 0 to 8.

So, multiple resonance peaks in the visible range are observed in the spectral visible range when generalized Thue–Morse multilayer is inserted between two periodical multilayer systems (Bragg mirrors). The transmission peaks can be made very sharp by increasing the reflectivity of the Bragg mirror and the repetition number P. In earlier works [10] we show that these results were obtained for the generalized Cantor multilayer structures. It appears that these properties are proper to the quasi-periodic multilayer structures.

4. Conclusions

The study into the spectral visible range of the juxtaposition of P multilayer systems constructed as: P generalized Thue–Morse multilayer $(\mathrm{GTM})^P$ for the triplet $(n, m, k) = (1, 2, 3)$, embedded between two periodical multilayer structures $H(LH)^J$, show the following results:

– Multiple resonance peaks in the visible range are observed when a generalized Thue–Morse multilayer is inserted between tow highly reflective mirrors;

– The number of peaks depends on the repetition number P of the generalized Thue–Morse-like multilayer;

– The transmission peaks can be made very sharp by increasing the reflectivity of the mirror and the repetition number P;

– The average of the full width at half maximum (FWHM) for each of the resonance peaks can be as narrow as possible with increasing P;

– If we use ON to present the high transmission and OFF the weak transmission for $\lambda = \lambda_0$, the light transmission coefficient T through the structure has the following switch-like property: S_0(ON)–S_1(OFF)–S_2(OFF)–S_3(ON)–S_4(ON)–S_5(OFF)–S_6(OFF)–...;

– Consequently we can use this optical device as a polychromatic filter or optical switching device.

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