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Statistical analysis of backscattered laser signals from rough targets in the laser altimeter application

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Statistical properties of the scattered field for a laser altimeter are considered. Analytical expressions are developed for the optical field correlation function and intensity correlation function, as well as the correlation degree of intensities of speckle patterns. The relationship between the backscattered signal intensity and the target roughness is analyzed.

Keywords: optical field, Gaussian random process, phase modulation factor.

1. Introduction

Topographic mapping of the earth, moon and other planets can be accomplished by employing a high power laser operating at high pulse repetition-rates with nanosecond pulse-width and micro-radian beam divergences. These systems are typically pointed at nadir and function by measuring the round-trip time of the laser pulses scattered from targets [1–3]. Under illumination of a laser beam, the received signal is a complex addition of de-phased but coherent wavelets coming from many independent scattering cells from the target surfaces. The received signal is disorderly distributed speckles instead of a uniform distribution of intensity. The performance of an optical energy-detecting receiver can be severely degraded by the speckle effects. The speckle properties have been studied extensively [4–11], but relatively little effort had been made to study target-induced speckle effects on the laser altimeter performance. To optimize system performance and sensor design, the statistics of the light wave scattering from random rough surfaces need to be analyzed. Random rough surfaces continue to attract great interest of researchers because of its broad applications in optics, radio wave propagation and remote sensing. Rapid progress towards characterizing rough surfaces has been made in recent years. A novel approach to modeling and classifying random rough surfaces for the evaluation of the electromagnetic wave scattering has been reported [12–14]. In this paper, the statistics of the rough target signature are investigated. The analytical expressions are derived for the optical field correlation function, and intensity correlation function, and the intensity covariance and correlation degree of speckle intensities. The roughness dependence of the signal strength at the receiver plane is analyzed.

2. Derivation of correlation function of the optical field

Consider a coordinate system, with $\mathbf{r} = (x, y)$ denoting a coordinate vector in the receiver plane, $\boldsymbol{\rho} = (\xi, \eta)$ representing the coordinate vector in the mean plane of the target surfaces. The receiver plane is parallel to the mean plane of target surfaces. Ignoring the time dependence of the optical field, the source amplitude distribution for a single-mode laser transmitter can be written as

$$U_0(\mathbf{r}) = u_0 \exp\left(-\frac{r^2}{w_0^2}\right) \tag{1}$$

where w_0 is the characteristic beam radius. Let L denote the distance between the target and the laser transmitter, and let k represent the wave number of the incident beam. Using the Huygens–Fresnel principle, the field distribution incident on the target may be written as

$$U_i(\boldsymbol{\rho}) = \frac{k\exp(jkL)}{2\pi jL}\int U_0(\mathbf{r})\exp\left(\frac{jk|\boldsymbol{\rho}-\mathbf{r}|^2}{2L}\right)\mathrm{d}^2\mathbf{r} \tag{2}$$

For convenience, atmospheric attenuation factor has been ignored in this expression, and it can be easily included in the final formulation. For an airborne laser altimeter, L is about several kilometers, and it is several hundred kilometers for a space-borne altimeter. The laser beam waist w_0 is of the order of millimeters. For a laser altimeter, the condition of Fraunhofer approximation is satisfied. In deriving optical field incident on the target, Fraunhofer approximation is considered. The field distribution on the mean plane of the target can be obtained and is written as

$$U_i(\boldsymbol{\rho}) = \frac{kw_0^2}{2jL}u_0\exp\left[jkL\left(1+\frac{\rho^2}{2L^2}\right)\right]\exp\left(-\frac{k^2w_0^2}{4L^2}\rho^2\right) \tag{3}$$

The divergence angle is of the order of milliradians for an airborne altimeter and even micro-radians for a space-based altimeter. The diameter of a laser footprint is the product of L and the laser beam divergence angle. It is evident that the term ρ^2/L^2 is extremely small and can be ignored. Since we are only concerned with the amplitude distribution of the optical field, the phase constant term can be omitted in Eq. (3). The optical field on the target plane can be written as

$$U_i(\boldsymbol{\rho}) = \frac{kw_0^2}{2L} u_0 \exp\left(-\frac{k^2 w_0^2}{4L^2}\rho^2\right) \tag{4}$$

The surface height fluctuations are modeled as a homogeneous, isotropic and zero-mean Gaussian random process. Let the target concerned be uniformly reflective at the scale of the laser foot print, with an amplitude reflection coefficient R. The electric field reflected from the target at the plane immediately adjacent to the scattering surface can be written as

$$U_r(\boldsymbol{\rho}) = R U_i(\boldsymbol{\rho}) \Phi(\boldsymbol{\rho}) \tag{5}$$

where $\Phi(\boldsymbol{\rho})$ is the phase modulation factor of the target surfaces, and is expressed as $\Phi(\boldsymbol{\rho}) = \exp[-j\phi(\boldsymbol{\rho})]$, and $\phi(\boldsymbol{\rho})$ is used to describe a random phase delay from point to point over target surfaces. Here, attention is restricted to the normal incident case. The phase change due to scattering from the target can be expressed as $\phi(\boldsymbol{\rho}) = 2kh(\boldsymbol{\rho})$, and $h(\boldsymbol{\rho})$ is the height fluctuation of the target. The variance of $\phi(\boldsymbol{\rho})$ is written as

$$\sigma_\phi^2 = 4k^2 \sigma_h^2 \tag{6}$$

where σ_h is the rms height of target surfaces and σ_ϕ is the rms phase induced by height fluctuations of target surfaces. The optical field at the receiver is obtained by applying the Huygens–Fresnel principle to the field at the target

$$U(\mathbf{r}) = \frac{k\exp(jkL)}{2\pi jL} \int U_r(\boldsymbol{\rho}) \exp\left(\frac{jk|\boldsymbol{\rho} - \mathbf{r}|^2}{2L}\right) \mathrm{d}^2\boldsymbol{\rho} \tag{7}$$

The correlation function of the speckle field at the receiver plane can be written as

$$\Gamma_U(\mathbf{r}_1, \mathbf{r}_2) = \langle U(\mathbf{r}_1) U^*(\mathbf{r}_2) \rangle =$$
$$= \left(\frac{kR}{2\pi L}\right)^2 \iint \mathrm{d}\boldsymbol{\rho}_1 \mathrm{d}\boldsymbol{\rho}_2 \, U_r(\boldsymbol{\rho}_1) U_r^*(\boldsymbol{\rho}_2) \exp\left[\frac{jk}{2L}\left(|\mathbf{r}_1 - \boldsymbol{\rho}_1|^2 - |\mathbf{r}_2 - \boldsymbol{\rho}_2|^2\right)\right] \tag{8}$$

where the angular brackets $\langle \; \rangle$ indicate ensemble averaging, and the symbol $*$ denotes the complex conjugation. As the rough surfaces are considered to be a zero-mean Gaussian random process, the joint-characteristic function is given by [15–17]

$$\langle \Phi(\boldsymbol{\rho}_1)\Phi^*(\boldsymbol{\rho}_2)\rangle = \exp\left\{-\sigma_\phi^2\left[1 - C(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)\right]\right\} \tag{9}$$

where $C(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)$ is the correlation function of the surface height variations. The correlation function of isotropic surfaces is usually assumed to be Gaussian [16, 17]

$$C(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1) = \exp\left(-\frac{|\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1|^2}{d^2}\right) \tag{10}$$

where d is the lateral correlation length of random height fluctuations. The joint-characteristic function is expanded into Taylor series

$$\langle \Phi(\boldsymbol{\rho}_1)\Phi^*(\boldsymbol{\rho}_2)\rangle = \exp(-\sigma_\phi^2)\sum_{n=0}^{\infty}\frac{\sigma_\phi^{2n}}{n!}\exp\left[-n\frac{|\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1|^2}{d^2}\right] \tag{11}$$

As the range L between the target and the transmitter is much larger than the diameter of the laser footprint, the following approximation has been made

$$\exp\left[\frac{jk}{2L}\left(|\mathbf{r}_1 - \boldsymbol{\rho}_1|^2 - |\mathbf{r}_2 - \boldsymbol{\rho}_2|^2\right)\right] \approx \exp\left[\frac{jk}{2L}(r_1^2 - r_2^2)\right]\exp\left[\frac{jk}{L}(\mathbf{r}_2\cdot\boldsymbol{\rho}_2 - \mathbf{r}_1\cdot\boldsymbol{\rho}_2)\right] \tag{12}$$

The correlation function of the optical field at the receiver plane is derived from Eq. (8), which is given by

$$\Gamma_U(\mathbf{r}_1, \mathbf{r}_2) = (\beta R u_0)^2\exp(-\sigma_\phi^2)\exp\left[-\frac{jk}{2L}(r_2^2 - r_1^2)\right]\exp\left[-\frac{1}{2w_0^2}|\mathbf{r}_2 - \mathbf{r}_1|^2\right]\times$$
$$\times\sum_{n=0}^{\infty}\frac{\sigma_\phi^{2n}}{(2n+\beta^2)n!}\exp\left[-\frac{\beta^2}{2(2n+\beta^2)}\frac{|\mathbf{r}_1 + \mathbf{r}_2|^2}{w_0^2}\right] \tag{13}$$

where $\beta = kw_0 d/2L$. The mean intensity at the receiver plane can be written as

$$\langle I(\mathbf{r})\rangle = \langle U(\mathbf{r})U^*(\mathbf{r})\rangle =$$
$$= (Ru_0\beta)^2\exp(-\sigma_\phi^2)\sum_{n=0}^{\infty}\frac{\sigma_\phi^{2n}}{(\beta^2+2n)n!}\exp\left[-\frac{2\beta^2}{\beta^2+2n}\frac{r^2}{w_0^2}\right] \tag{14}$$

Here, we limit our attention to the targets whose irregularities are much larger than the incident wavelength, so σ_ϕ is much larger than one. This implies that significant contributions of joint-characteristic function to the integral of Eq. (8) exist only for very small values of $|\rho_2 - \rho_1|$. The integrand of Eq. (8) is very well approximated retaining only first two terms in the Taylor series of Eq. (10)

$$C(\rho_2 - \rho_1) \cong 1 - \frac{|\rho_2 - \rho_1|^2}{d^2} \tag{15}$$

Using Eq. (15), the correlation function of the speckle field at the receiver plane can be simplified. Using Eq. (8), we obtain

$$\Gamma_U(\mathbf{r}_1, \mathbf{r}_2) = \frac{(\beta R u_0)^2}{\beta^2 + 2\sigma_\phi^2} \exp\left[-\frac{jk(r_2^2 - r_1^2)}{2L} - \frac{|\mathbf{r}_2 - \mathbf{r}_1|^2}{2w_0^2}\right] \times$$
$$\times \exp\left[-\frac{\beta^2}{\beta^2 + 2\sigma_\phi^2} \frac{|\mathbf{r}_2 + \mathbf{r}_1|^2}{2w_0^2}\right] \tag{16}$$

The mean intensity at the receiver plane is written as

$$\langle I(\mathbf{r})\rangle = \Gamma_U(\mathbf{r}, \mathbf{r}) = \frac{(\beta R u_0)^2}{\beta^2 + 2\sigma_\phi^2} \exp\left(-\frac{2\beta^2}{\beta^2 + 2\sigma_\phi^2} \frac{r^2}{w_0^2}\right) \tag{17}$$

From the data listed of soil [18], the ratio of the lateral autocorrelation length d of the soil surface to the rms height σ_h is less than 30. So, for the air-borne or space-borne laser altimeter, the following condition is satisfied

$$\frac{\sigma_\phi}{\beta} = 4 \frac{\sigma_h}{d} \frac{L}{w_0} \gg 1 \tag{18}$$

The mean intensity at the receiver plane is simplified to

$$\langle I(\mathbf{r})\rangle = \frac{I_0}{32} R^2 \left(\frac{w_0}{L}\right)^2 \left(\frac{d}{\sigma_h}\right)^2 \exp\left[-\frac{1}{16}\left(\frac{d}{\sigma_h}\right)^2 \frac{r^2}{L^2}\right] \tag{19}$$

where $I_0 = u_0^2$. Equation (19) indicates that the intensity distribution on the receiver plane is closely related to the lateral correlation length and the rms height of target surfaces.

The target roughness is characterized by the parameter (σ_h/d). The rms height of the smooth target is relatively small compared with that of the rough target, while its lateral correlation length is large in comparison with that of the rough target. It can be easily seen that the intensity at the receiver plane decreases rapidly with the increase

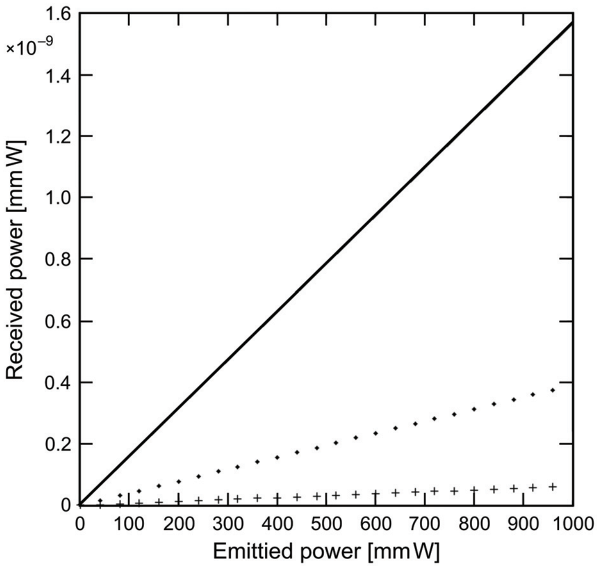

Figure. Power received at the centre of the aperture versus the emitted power. $L = 1000$ m, $w_0 = 1$ mm, $R = 0.7$, σ_h/d ratio for solid line, dotted line and + line are 0.1, 0.2 and 0.5, respectively.

of the target roughness. Targets will approach Lambertian surfaces with roughness increasing. The Figure shows the dependence of the intensity on the target roughness. The aperture will collect more scattered laser energy from a smooth target than from a rough one; hence the signal-to-noise ratio of laser radar is higher for a smooth surface than for a rough one. This implies that a rough target will reflect the incident wave more uniformly in all possible directions in the hemisphere than a smooth target. The target roughness has significant influence on the laser radar performance, with performance deteriorating as targets become rough.

3. Derivation of intensity correlation function and intensity covariance

The correlation function of intensity is written as

$$\Gamma_I(\mathbf{r_1}, \mathbf{r_2}) = \langle I(\mathbf{r_1}) I(\mathbf{r_2}) \rangle \tag{20}$$

We restrict our attention here to fully developed speckle patterns produced by rough targets. The speckle field obeys the classical circular Gaussian statistics. The intensity correlation function at the receiver plane can be expressed in terms of the correlation function of the speckle field

$$\Gamma_I(\mathbf{r_1}, \mathbf{r_2}) = \langle I(\mathbf{r_1}) I(\mathbf{r_2}) \rangle = \langle I(\mathbf{r_1}) \rangle \langle I(\mathbf{r_2}) \rangle + |\Gamma_U(\mathbf{r_1}, \mathbf{r_2})|^2 \tag{21}$$

Using Eqs. (16) and (17), we obtain

$$\Gamma_I(\mathbf{r_1}, \mathbf{r_2}) = \frac{(\beta u_0 R)^4}{(\beta^2 + 2\sigma_\phi^2)^2} \exp\left(-\frac{2\beta^2}{\beta^2 + 2\sigma_\phi^2}\,\frac{r_1^2 + r_2^2}{w_0^2}\right) + $$
$$+ \frac{(\beta u_0 R)^4}{(\beta^2 + 2\sigma_\phi^2)^2} \exp\left(-\frac{|\mathbf{r_2} - \mathbf{r_1}|^2}{w_0^2}\right) \exp\left(-\frac{\beta^2}{\beta^2 + 2\sigma_\phi^2}\,\frac{|\mathbf{r_1} + \mathbf{r_2}|^2}{w_0^2}\right) \tag{22}$$

The covariance of intensity is defined as

$$C_I(\mathbf{r_1}, \mathbf{r_2}) = \langle I(\mathbf{r_1})\, I(\mathbf{r_2})\rangle - \langle I(\mathbf{r_1})\rangle \langle I(\mathbf{r_2})\rangle \tag{23}$$

The covariance of intensity at the receiver plane can be written as

$$C_I(\mathbf{r_1}, \mathbf{r_2}) = \frac{(\beta u_0 R)^4}{(\beta^2 + 2\sigma_\phi^2)^2} \exp\left(-\frac{|\mathbf{r_2} - \mathbf{r_1}|^2}{w_0^2}\right) \exp\left(-\frac{\beta^2}{\beta^2 + 2\sigma_\phi^2}\,\frac{|\mathbf{r_1} + \mathbf{r_2}|^2}{w_0^2}\right) \tag{24}$$

The variance of intensity is easily derived

$$\sigma_I^2 = C_I(\mathbf{r}, \mathbf{r}) = \frac{(\beta u_0 R)^4}{(\beta^2 + 2\sigma_\phi^2)^2} \exp\left(-\frac{4\beta^2}{\beta^2 + 2\sigma_\phi^2}\,\frac{r^2}{w_0^2}\right) = \langle I(\mathbf{r})\rangle^2 \tag{25}$$

So, we can see that the standard deviation σ_I of polarized speckle patterns is equal to the mean intensity. The contrast of a speckle pattern is the ratio $\sigma_I/\langle I\rangle$, and it is evident that the contrast of speckle patterns is unity. The conclusion is consistent with the result derived from the theory of probability [6].

Consider the correlation degree γ_{12} of the speckle intensities I_n of two different speckle patterns, which is defined by

$$\gamma_{12} = \frac{\langle I_1 I_2\rangle - \langle I_1\rangle\langle I_2\rangle}{\left[\left(\langle I_1^2\rangle - \langle I_1\rangle^2\right)\left(\langle I_2^2\rangle - \langle I_2\rangle^2\right)\right]^{1/2}} \tag{26}$$

Using Eq. (19), the correlation degree of the Speckle intensities is written as

$$\gamma_{12} = \exp\left(-\frac{\Delta r^2}{w_0^2}\right) \tag{27}$$

Equation (27) indicates that the correlation degree of intensities of two different patterns is independent of the target roughness and only determined by the laser beam waist.

4. The average speckle size

The average size of a speckle is an important parameter, which has decisive effect on the performance of laser altimeters. The speckle size can be estimated by the width of the spatial correlation function of intensity. To calculate the speckle size, it is convenient to work with the complex coherence factor $\mu(\Delta x, \Delta y)$, which is defined by

$$\mu(\Delta x, \Delta y) = \frac{\Gamma_U(\mathbf{r_1}, \mathbf{r_2})}{\left[\Gamma_U(\mathbf{r_1}, \mathbf{r_1}) \cdot \Gamma_U(\mathbf{r_2}, \mathbf{r_2})\right]^{1/2}} \tag{28}$$

The correlation area of the speckle is defined as [6]

$$S_C = \iint_{-\infty}^{+\infty} |\mu(\Delta x, \Delta y)|^2 \mathrm{d}\Delta x \mathrm{d}\Delta y \tag{29}$$

Using Eq. (16) and Eq. (28), we obtain

$$\mu(\Delta x, \Delta y) = \sqrt{\gamma_{12}} \tag{30}$$

Substituting Eq. (30) into Eq. (29) and integrating, we get the correlation area of a speckle

$$S_C = \pi w_0^2 \left(1 + \frac{\beta^2}{2\sigma_\phi^2}\right) \approx \pi w_0^2 \tag{31}$$

Assuming speckles to be circular, the average correlation radius of speckles is approximately equal to w_0. The above relation indicates that the speckle radius is equal to the beam waist for the far field situation.

5. Conclusions

Statistical properties of signal returns from remotely rough surfaces under single mode laser illumination have been investigated. The analytical expressions of the optical field and intensity correlation functions are presented. The results show that the mean signal intensity at the receiver plane is closely related to the target roughness, and the average speckle size and the correlation degree of the speckle intensities of two different patterns are independent of the target roughness.

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