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Electron transport in the multi-terminal quantum dot system

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The time-dependent electron transport through a multi-terminal quantum dot system is studied. External microwave fields with arbitrary amplitudes, phases and frequencies are applied to different parts of the system considered. The dependence of the average current and average differential conductance on different parameters of the external microwave fields is analyzed. Special attention is paid to the photon–electron pump effect observed for some values of the quantum dot system parameters.

Keywords: photon assisted tunneling, quantum dot.

1. Introduction

The development of experimental techniques on a nanometer scale has stirred up interest in the electron transport in the quantum dot systems. Some interest has been focused on the transport properties of a quantum dot (QD) under influence of external microwave (MW) fields applied to different parts of the system considered. New effects have been observed and theoretically described, *e.g.*, photon-assisted tunneling [1, 2], photon-electron pumps [2], and others. In most investigations, the current flow through a QD placed between two leads was considered, *e.g.*, [3–6]. Here, we study a QD coupled with N leads, N being an arbitrary number, as such systems can be useful ingredients in the field of spintronics and quantum computations. The MW fields applied to the QD and leads have different phases, amplitudes and frequencies. Our work generalizes that of SUN and LIN [7] to the case of the multi-terminal QD. The dependence of the time-averaged current $\langle j \rangle$ and the differential conductance $\partial\langle j \rangle / \partial\mu$ on different parameters of the external MW fields is considered.

In the next section, we present a model and give final formulas for average current and differential conductance. The last section includes the results of numerical calculations of the current and conductance and a short discussion together with a summary.

2. Theoretical approach

Let us consider a system composed of the QD coupled through the tunneling barriers with N metallic leads described by the Hamiltonian:

$$H = \sum_{i=1}^{N}\sum_{\mathbf{k_i}}\varepsilon_{\mathbf{k_i}}(t)c^+_{\mathbf{k_i}}c_{\mathbf{k_i}} + \varepsilon_d(t)c^+_d c_d + \sum_{i=1}^{N}\sum_{\mathbf{k_i}}V_{\mathbf{k_i}d}\,c^+_{\mathbf{k_i}}c_d + \text{h.c.} \tag{1}$$

For simplicity, the dot is characterized only by the single level ε_d and we have neglected the intra-dot electron–electron interactions. Here, c^+_d (c_d) creates (destroys) an electron in the QD, $c^+_{\mathbf{k_i}}$ ($c_{\mathbf{k_i}}$) creates (destroys) an electron with the momentum **k** in the *i*-th lead and $V_{\mathbf{k_i}d}$ denotes the coupling between the QD and the *i*-th lead. We consider our mesoscopic system in the presence of the external MW fields applied to the QD and leads, and we assume $\varepsilon_{\mathbf{k_i}}(t) = \varepsilon_{\mathbf{k_i}} + \Delta_i\cos(\omega_i t + \varphi_i)$, $\varepsilon_d(t) = \varepsilon_d + \Delta_d\cos(\omega_d t + \varphi_d)$. The amplitudes, phases and frequencies of the MW fields can be chosen arbitrarily. Applying the formalism of the nonequilibrium Green's functions [2, 3] we can derive the formula for the time-averaged current flowing into the QD from the α-th lead in the form:

$$\langle j_\alpha\rangle = -\frac{e}{\hbar}\frac{\Gamma_\alpha}{2\pi}\Bigg\{\sum_{j=1}^{N}\Gamma_j\sum_{nmpl}J_n\left(\frac{\Delta_d}{\omega_d}\right)J_m\left(\frac{\Delta_j}{\omega_j}\right)J_l\left(\frac{\Delta_d}{\omega_d}\right)J_p\left(\frac{\Delta_j}{\omega_j}\right)$$

$$\times\exp\Big[i(p-m)\varphi_j\Big]\,\delta\Big((n-l)\omega_d + (p-m)\omega_j\Big)\int \mathrm{d}x\, f_\alpha(x)F_j(x)$$

$$+\,2\sum_{nmpl}J_n\left(\frac{\Delta_d}{\omega_d}\right)J_m\left(\frac{\Delta_\alpha}{\omega_\alpha}\right)J_l\left(\frac{\Delta_d}{\omega_d}\right)J_p\left(\frac{\Delta_\alpha}{\omega_\alpha}\right)\delta\Big((n-l)\omega_d + (p-m)\omega_\alpha\Big)$$

$$\times\left[\sin\Big(\varphi_\alpha(p-m)\Big)\int\mathrm{d}x\, f_\alpha(x)G_\alpha(x) - \cos\Big(\varphi_\alpha(p-m)\Big)\frac{\Gamma}{2}\int\mathrm{d}x\, f_\alpha(x)F_\alpha(x)\right]\Bigg\} \tag{2}$$

where:

$$F_\alpha(x) = \left[\Big(x - \varepsilon_d - n\omega_d + m\omega_\alpha\Big)^2 + \left(\frac{\Gamma}{2}\right)^2\right]^{-1},$$

$$G_\alpha(x) \;=\; F_\alpha(x)\Big(x - \varepsilon_d - n\,\omega_d + m\,\omega_\alpha\Big).$$

Here we use the wide-band limit, $\Gamma_j \;=\; 2\pi\sum_{\mathbf{k_j}}\left|V_{\mathbf{k_j}d}\right|^2\delta(\varepsilon-\varepsilon_{\mathbf{k_j}}),\ \Gamma \;=\; \sum_j \Gamma_j,\ \ f_\alpha(x)$ is the Fermi distribution function of electrons in the α-th lead and J_n denotes the Bessel function of the first kind.

The important characteristic of the QD system is the differential conductance $\partial\langle j_\alpha\rangle/\partial\mu_\beta$. In our model, we obtain, *e.g.*, for $\alpha = \beta$, the following formula (for zero temperature and ω_α/ω_d = *integer* ≥ 1):

$$\begin{aligned}\frac{\partial\langle j_\alpha\rangle}{\partial\mu_\alpha} \;=\; -\frac{e}{\hbar}\sum_{nmp} J_n\!\left(\frac{\Delta_d}{\omega_d}\right) J_m\!\left(\frac{\Delta_\alpha}{\omega_\alpha}\right) J_p\!\left(\frac{\Delta_\alpha}{\omega_\alpha}\right) J_{n-(m-p)\,\omega_\alpha/\omega_d}\!\left(\frac{\Delta_d}{\omega_d}\right) \\ \times\left\{\frac{\Gamma_\alpha^2}{2\pi}\exp\Big[i(p-m)\,\varphi_\alpha\Big]F_\alpha(\mu_\alpha)\right. \\ \left. +\,\frac{\Gamma_\alpha}{2\pi}\left[\sin\Big(\varphi_\alpha(p-m)\Big)\,G_\alpha(\mu_\alpha) - \frac{\Gamma}{2}\cos\Big(\varphi_\alpha(p-m)\Big)\,F_\alpha(\mu_\alpha)\right]\right\}.\end{aligned} \tag{3}$$

For $\alpha \neq \beta$ or for the case where ω_α/ω_d is not a simple integer ratio, the corresponding formulas are slightly different (not given here).

3. Results and discussion

On the basis of the average current and differential conductance formulas, Eqs. (2) and (3), we study numerically both the current and conductance of the QD coupled with several leads in the presence of the MW fields. In calculations all energies are given in the units of Γ_1 ($\Gamma_1 = \Gamma_2 = ... = \Gamma_N = 1$), the frequency, the current and its derivatives are given in $\Gamma_1/\hbar$, $e\Gamma_1/\hbar$ and $e/\hbar$ units, respectively.

In Figure 1, we present the results for $\partial\langle j_1\rangle/\partial\mu_1$ *vs.* μ_1 for the case of the QD coupled with different numbers of leads. Note that accordingly to Eq. (3) the differential conductance $\partial\langle j_1\rangle/\partial\mu_1$ depends only on the characteristics of the QD and the first lead, *i.e.*, on such parameters as Δ_1, Δ_d, φ_1, ω_d, ω_1 and μ_1, and does not depend on the amplitudes Δ_i, the phases φ_i and the frequencies ω_i describing other leads. We assumed the frequencies ω_1 and ω_d to be equal and the panels **a**, **b** and **c** give the results for different phases φ_1. In each case, one can observe the peaks localized at $k\omega_d$ (= $k\omega_1$) with the distance between them equal ω_d (= ω_1). The main effect of the increasing number of leads coupled with the QD is the broadening of the corresponding peaks of the conductance. For comparison, in Fig. 2 we show the results for different frequencies of the MW fields acting on the first lead and the QD, $\omega_1 = 8$,

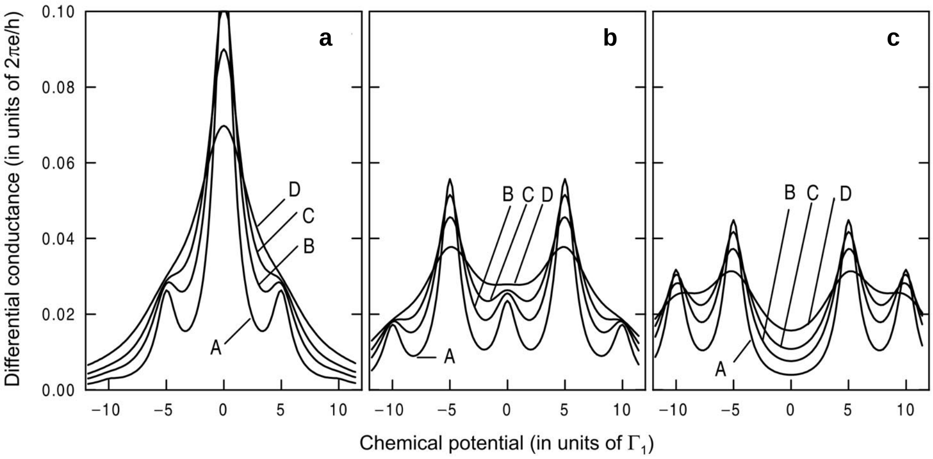

Fig. 1. Differential conductance $\partial\langle j_1\rangle/\partial\mu_1$ *vs.* chemical potential μ_1 for different numbers N of the leads coupled with the QD (curves *A*, *B*, *C* and *D* correspond to $N = 2$, 3, 4 and 6, respectively), $\Delta_1 = 8$, $\Delta_d = 4$, $\omega_1 = \omega_d = 5$, $\varepsilon_d = 0$, $\Gamma_1 = \Gamma_2 = ... = \Gamma_N = 1$. The parts **a**, **b** and **c** correspond to different phases of the MW field in the first lead, $\varphi_1 = 0$, π/2 and π, respectively. The phases of the MW fields in other leads are equal to zero.

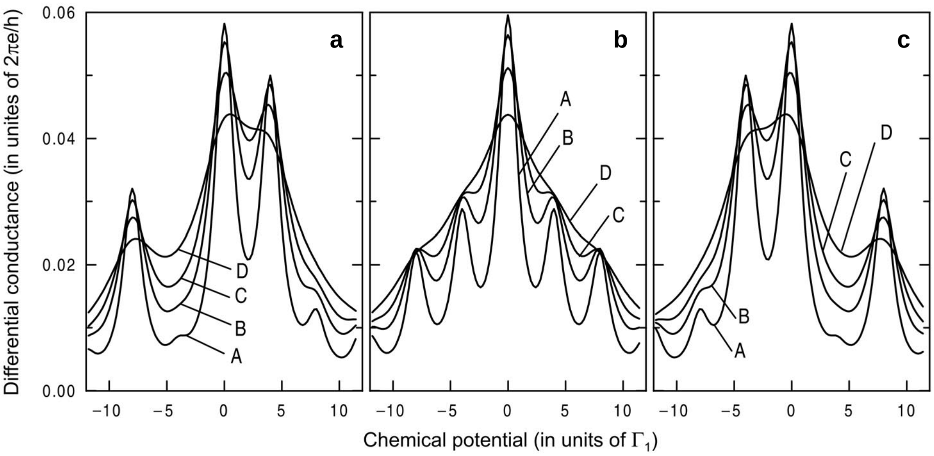

Fig. 2. The same as in Fig. 1 but for $\omega_1 = 8$ and $\omega_d = 4$.

$\omega_d = 4$. Now one can observe the peaks of the conductance localized not only at $k\omega_d$ but also at $k\omega_d \pm n\omega_1$.

In Figure 3, we present the results for the current $\langle j_1\rangle$ flowing from one of the leads (here numbered as the first) in the case of vanishing dc bias voltage between leads (photon–electron pump effect). The non-zero average current is induced by a cyclic adiabatic change of the parameters describing the QD system. In Fig. 3**a**, we consider the QD coupled with two leads and the electron pumping is achieved assuming

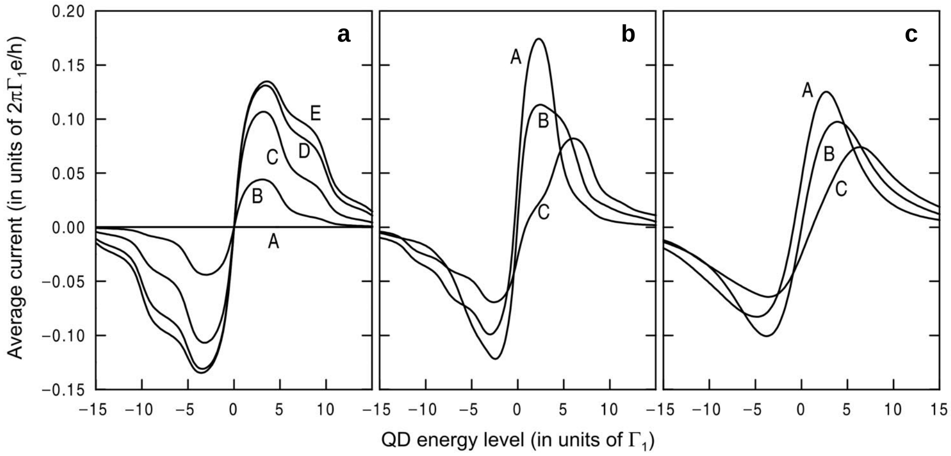

Fig. 3. Averaged current $\langle j_1 \rangle$ *vs.* ε_d in the case of vanishing dc bias voltage between leads for the QD coupled with two (parts **a**, **b**) and six leads (part **c**). **a** – the curves *A*, *B*, *C*, *D* and *E* correspond to $\varphi_1 = 0$, $\pi/4$, $\pi/2$, $3\pi/4$ and π, respectively, ($\Delta_1 = 8$, $\Delta_d = 4$, $\Delta_2 = 0$, $\omega_1 = \omega_d = 5$); **b** – the curves *A*, *B* and *C* correspond to $(\omega_1, \omega_d) = (4, 8)$, $(1, 8)$ and $(8, 4)$, respectively, ($\Delta_1 = 8$, $\Delta_d = 4$, $\Delta_2 = 0$, $\varphi_1 = 0$); **c** – the same as in Fig. 3**b** but for the QD coupled with six leads for $\Delta_1 = 8$, $\Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 0$.

different phases of the MW fields acting on the first lead and QD. For the parameters taken in our calculations the current is the antisymmetric function of ε_d and increases with the increasing value of the phase difference $\varphi_1 - \varphi_d$. The pump current through the QD can also be obtained taking different frequencies of the MW fields applied to different parts of the QD system – see the results in Fig. 3**b**. The effect of the increasing number of leads coupled with the QD on the pump current is visible comparing the results given in Figs. 3**b** and 3**c** and can be explained analyzing the formula for the current, Eq. (2).

Summing up, we have used the nonequilibrium Green's functions method to study the electron tunneling through the QD coupled with *N* leads in the presence of external MW field. We have analyzed the average current and conductance in the case when the MW fields applied to different parts of the system have different phases, amplitudes and frequencies. The conductance exhibits a very rich structure with the peaks corresponding to the multiple photon assisted tunneling. We also studied the photon electron pump current due to different frequencies or phases of the applied MW fields.

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